Mid Term Syllabus

RF AND MICROWAVE ENGINEERING

Lecture 1

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Contents:

- Introduction to Microwave Engineering
- Maxwell Equations
- Fields in Media and Boundary Conditions
- The Helmholtz Equation of Plane Waves in Lossless
- Lossy and Good Conductor Medium

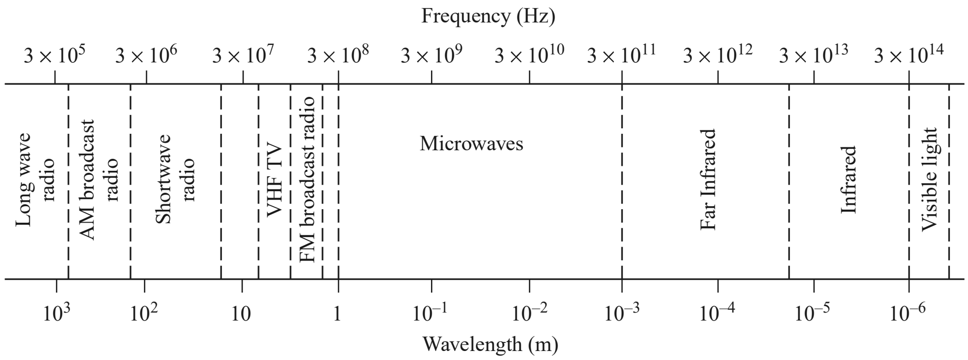
Chapter One

Electromagnetic Theory

We begin our study of microwave engineering with a brief overview of the history and major applications of microwave technology, followed by a review of some of the fundamental topics in electromagnetic theory that we will need throughout the book. Further discussion of these topics may be found in references [1–8].

1.1 INTRODUCTION TO MICROWAVE ENGINEERING

The field of radio frequency (RF) and microwave engineering generally covers the behavior of alternating current signals with frequencies in the range of 100 MHz (1 MHz = 10^6 Hz) to 1000 GHz (1 GHz = 10^9 Hz). RF frequencies range from very high frequency (VHF) (30–300 MHz) to ultra high frequency (UHF) (300–3000 MHz), while the term *microwave* is typically used for frequencies between 3 and 300 GHz, with a corresponding electrical wavelength between $\lambda = c/f = 10$ cm and $\lambda = 1$ mm, respectively. Signals with wavelengths on the order of millimeters are often referred to as *millimeter waves*. Figure 1.1 shows the location of the RF and microwave frequency bands in the electromagnetic spectrum. Because of the high frequencies (and short wavelengths), standard circuit theory often cannot be used directly to solve microwave network problems. In a sense, standard circuit theory is an approximation, or special case, of the broader theory of electromagnetics as described by Maxwell's equations. This is due to the fact that, in general, the lumped circuit element approximations of circuit theory may not be valid at high RF and microwave frequencies. Microwave components often act as *distributed elements*, where the phase of the voltage or current changes significantly over the physical extent of the device because the device dimensions are on the order of the electrical wavelength. At much lower frequencies the wavelength is large enough that there is insignificant phase variation across the dimensions of a component. The other extreme of frequency can be identified as optical engineering, in which the wavelength is much shorter than the dimensions of the component. In this case Maxwell's equations can be simplified to the geometrical optics regime, and optical systems can be designed with the theory of geometrical optics. Such



Typical Frequencies	
AM broadcast band	535–1605 kHz
Short wave radio band	3–30 MHz
FM broadcast band	88–108 MHz
VHF TV (2–4)	54–72 MHz
VHF TV (5–6)	76–88 MHz
UHF TV (7–13)	174–216 MHz
UHF TV (14–83)	470–890 MHz
US cellular telephone	824–849 MHz
	869–894 MHz
European GSM cellular	880–915 MHz
	925–960 MHz
GPS	1575.42 MHz
	1227.60 MHz
Microwave ovens	2.45 GHz
US DBS	11.7–12.5 GHz
US ISM bands	902–928 MHz
	2.400–2.484 GHz
	5.725–5.850 GHz
US UWB radio	3.1–10.6 GHz

Approximate Band Designations	
Medium frequency	300 kHz–3 MHz
High frequency (HF)	3 MHz–30 MHz
Very high frequency (VHF)	30 MHz–300 MHz
Ultra high frequency (UHF)	300 MHz–3 GHz
L band	1–2 GHz
S band	2–4 GHz
C band	4–8 GHz
X band	8–12 GHz
Ku band	12–18 GHz
K band	18–26 GHz
Ka band	26–40 GHz
U band	40–60 GHz
V band	50–75 GHz
E band	60–90 GHz
W band	75–110 GHz
F band	90–140 GHz

FIGURE 1.1 The electromagnetic spectrum.

techniques are sometimes applicable to millimeter wave systems, where they are referred to as *quasi-optical*.

In RF and microwave engineering, then, one must often work with Maxwell's equations and their solutions. It is in the nature of these equations that mathematical complexity arises since Maxwell's equations involve vector differential or integral operations on vector field quantities, and these fields are functions of spatial coordinates. One of the goals of this book is to try to reduce the complexity of a field theory solution to a result that can be expressed in terms of simpler circuit theory, perhaps extended to include distributed elements (such as transmission lines) and concepts (such as reflection coefficients and scattering parameters). A field theory solution generally provides a complete description of the electromagnetic field at every point in space, which is usually much more information than we need for most practical purposes. We are typically more interested in terminal quantities such as power, impedance, voltage, and current, which can often be expressed in terms of these extended circuit theory concepts. It is this complexity that adds to the challenge, as well as the rewards, of microwave engineering.

Applications of Microwave Engineering

Just as the high frequencies and short wavelengths of microwave energy make for difficulties in the analysis and design of microwave devices and systems, these same aspects provide unique opportunities for the application of microwave systems. The following considerations can be useful in practice:

- Antenna gain is proportional to the electrical size of the antenna. At higher frequencies, more antenna gain can be obtained for a given physical antenna size, and this has important consequences when implementing microwave systems.
- More bandwidth (directly related to data rate) can be realized at higher frequencies. A 1% bandwidth at 600 MHz is 6 MHz, which (with binary phase shift keying modulation) can provide a data rate of about 6 Mbps (megabits per second), while at 60 GHz a 1% bandwidth is 600 MHz, allowing a 600 Mbps data rate.
- Microwave signals travel by line of sight and are not bent by the ionosphere as are lower frequency signals. Satellite and terrestrial communication links with very high capacities are therefore possible, with frequency reuse at minimally distant locations.
- The effective reflection area (radar cross section) of a radar target is usually proportional to the target's electrical size. This fact, coupled with the frequency characteristics of antenna gain, generally makes microwave frequencies preferred for radar systems.
- Various molecular, atomic, and nuclear resonances occur at microwave frequencies, creating a variety of unique applications in the areas of basic science, remote sensing, medical diagnostics and treatment, and heating methods.

The majority of today's applications of RF and microwave technology are to wireless networking and communications systems, wireless security systems, radar systems, environmental remote sensing, and medical systems. As the frequency allocations listed in Figure 1.1 show, RF and microwave communications systems are pervasive, especially today when wireless connectivity promises to provide voice and data access to "anyone, anywhere, at any time."

Modern wireless telephony is based on the concept of *cellular frequency reuse*, a technique first proposed by Bell Labs in 1947 but not practically implemented until the 1970s. By this time advances in miniaturization, as well as increasing demand for wireless communications, drove the introduction of several early cellular telephone systems in Europe, the United States, and Japan. The *Nordic Mobile Telephone* (NMT) system was deployed in 1981 in the Nordic countries, the *Advanced Mobile Phone System* (AMPS) was introduced in the United States in 1983 by AT&T, and NTT in Japan introduced its first mobile phone service in 1988. All of these early systems used analog FM modulation, with their allocated frequency bands divided into several hundred narrow band voice channels. These early systems are usually referred to now as *first-generation* cellular systems, or 1G.

Second-generation (2G) cellular systems achieved improved performance by using various digital modulation schemes, with systems such as GSM, CDMA, DAMPS, PCS, and PHS being some of the major standards introduced in the 1990s in the United States, Europe, and Japan. These systems can handle digitized voice, as well as some limited data, with data rates typically in the 8 to 14 kbps range. In recent years there has been a wide variety of new and modified standards to transition to handheld services that include voice, texting, data networking, positioning, and Internet access. These standards are variously known as 2.5G, 3G, 3.5G, 3.75G, and 4G, with current plans to provide data rates up to at least 100 Mbps. The number of subscribers to wireless services seems to be keeping pace with the growing power and access provided by modern handheld wireless devices; as of 2010 there were more than five billion cell phone users worldwide.

Satellite systems also depend on RF and microwave technology, and satellites have been developed to provide cellular (voice), video, and data connections worldwide. Two large satellite constellations, Iridium and Globalstar, were deployed in the late 1990s to provide worldwide telephony service. Unfortunately, these systems suffered from both technical drawbacks and weak business models and have led to multibillion dollar financial failures. However, smaller satellite systems, such as the Global Positioning Satellite (GPS) system and the Direct Broadcast Satellite (DBS) system, have been extremely successful.

Wireless local area networks (WLANs) provide high-speed networking between computers over short distances, and the demand for this capability is expected to remain strong. One of the newer examples of wireless communications technology is *ultra wide band* (UWB) radio, where the broadcast signal occupies a very wide frequency band but with a very low power level (typically below the ambient radio noise level) to avoid interference with other systems.

Radar systems find application in military, commercial, and scientific fields. Radar is used for detecting and locating air, ground, and seagoing targets, as well as for missile guidance and fire control. In the commercial sector, radar technology is used for air traffic control, motion detectors (door openers and security alarms), vehicle collision avoidance, and distance measurement. Scientific applications of radar include weather prediction, remote sensing of the atmosphere, the oceans, and the ground, as well as medical diagnostics and therapy. Microwave radiometry, which is the passive sensing of microwave energy emitted by an object, is used for remote sensing of the atmosphere and the earth, as well as in medical diagnostics and imaging for security applications.

A Short History of Microwave Engineering

Microwave engineering is often considered a fairly mature discipline because the fundamental concepts were developed more than 50 years ago, and probably because radar, the first major application of microwave technology, was intensively developed as far back as World War II. However, recent years have brought substantial and continuing developments in high-frequency solid-state devices, microwave integrated circuits, and computer-aided design techniques, and the ever-widening applications of RF and microwave technology to wireless communications, networking, sensing, and security have kept the field active and vibrant.

The foundations of modern electromagnetic theory were formulated in 1873 by James Clerk Maxwell, who hypothesized, solely from mathematical considerations, electromagnetic wave propagation and the idea that light was a form of electromagnetic energy. Maxwell's formulation was cast in its modern form by Oliver Heaviside during the period from 1885 to 1887. Heaviside was a reclusive genius whose efforts removed many of the mathematical complexities of Maxwell's theory, introduced vector notation, and provided a foundation for practical applications of guided waves and transmission lines. Heinrich Hertz, a German professor of physics and a gifted experimentalist who understood the theory published by Maxwell, carried out a set of experiments during the period 1887–1891 that validated Maxwell's theory of electromagnetic waves. Figure 1.2 is a photograph of the original equipment used by Hertz in his experiments. It is interesting to observe that this is an instance of a discovery occurring after a prediction has been made on theoretical grounds—a characteristic of many of the major discoveries throughout the history of science. All of the practical applications of electromagnetic theory—radio, television, radar, cellular telephones, and wireless networking—owe their existence to the theoretical work of Maxwell.

Because of the lack of reliable microwave sources and other components, the rapid growth of radio technology in the early 1900s occurred primarily in the HF to VHF range. It was not until the 1940s and the advent of radar development during World War II that microwave theory and technology received substantial interest. In the United States, the Radiation Laboratory was established at the Massachusetts Institute of Technology to develop radar theory and practice. A number of talented scientists, including N. Marcuvitz,

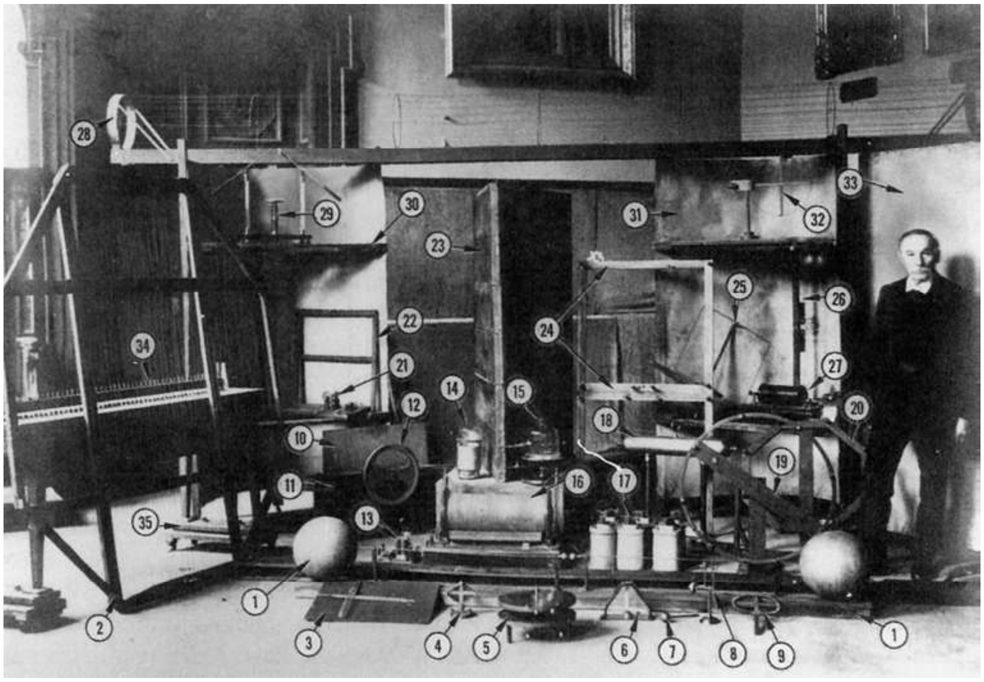


FIGURE 1.2 Original apparatus used by Hertz for his electromagnetics experiments. (1) 50 MHz transmitter spark gap and loaded dipole antenna. (2) Wire grid for polarization experiments. (3) Vacuum apparatus for cathode ray experiments. (4) Hot-wire galvanometer. (5) Reiss or Knochenhauer spirals. (6) Rolled-paper galvanometer. (7) Metal sphere probe. (8) Reiss spark micrometer. (9) Coaxial line. (10–12) Equipment to demonstrate dielectric polarization effects. (13) Mercury induction coil interrupter. (14) Meidinger cell. (15) Bell jar. (16) Induction coil. (17) Bunsen cells. (18) Large-area conductor for charge storage. (19) Circular loop receiving antenna. (20) Eight-sided receiver detector. (21) Rotating mirror and mercury interrupter. (22) Square loop receiving antenna. (23) Equipment for refraction and dielectric constant measurement. (24) Two square loop receiving antennas. (25) Square loop receiving antenna. (26) Transmitter dipole. (27) Induction coil. (28) Coaxial line. (29) High-voltage discharger. (30) Cylindrical parabolic reflector/receiver. (31) Cylindrical parabolic reflector/transmitter. (32) Circular loop receiving antenna. (33) Planar reflector. (34, 35) Battery of accumulators. Photographed on October 1, 1913, at the Bavarian Academy of Science, Munich, Germany, with Hertz's assistant, Julius Amman.

Photograph and identification courtesy of J. H. Bryant.

I. I. Rabi, J. S. Schwinger, H. A. Bethe, E. M. Purcell, C. G. Montgomery, and R. H. Dicke, among others, gathered for a very intensive period of development in the microwave field. Their work included the theoretical and experimental treatment of waveguide components, microwave antennas, small-aperture coupling theory, and the beginnings of microwave network theory. Many of these researchers were physicists who returned to physics research after the war, but their microwave work is summarized in the classic 28-volume Radiation Laboratory Series of books that still finds application today.

Communications systems using microwave technology began to be developed soon after the birth of radar, benefiting from much of the work that was originally done for radar systems. The advantages offered by microwave systems, including wide bandwidths and line-of-sight propagation, have proved to be critical for both terrestrial and satellite communications systems and have thus provided an impetus for the continuing development of low-cost miniaturized microwave components. We refer the interested reader to references [1] and [2] for further historical perspectives on the fields of wireless communications and microwave engineering.

1.2 MAXWELL'S EQUATIONS

Electric and magnetic phenomena at the macroscopic level are described by Maxwell's equations, as published by Maxwell in 1873. This work summarized the state of electromagnetic science at that time and hypothesized from theoretical considerations the existence of the electrical displacement current, which led to the experimental discovery by Hertz of electromagnetic wave propagation. Maxwell's work was based on a large body of empirical and theoretical knowledge developed by Gauss, Ampere, Faraday, and others. A first course in electromagnetics usually follows this historical (or deductive) approach, and it is assumed that the reader has had such a course as a prerequisite to the present material. Several references are available [3–7] that provide a good treatment of electromagnetic theory at the undergraduate or graduate level.

This chapter will outline the fundamental concepts of electromagnetic theory that we will require later in the book. Maxwell's equations will be presented, and boundary conditions and the effect of dielectric and magnetic materials will be discussed. Wave phenomena are of essential importance in microwave engineering, and thus much of the chapter is spent on topics related to plane waves. Plane waves are the simplest form of electromagnetic waves and so serve to illustrate a number of basic properties associated with wave propagation. Although it is assumed that the reader has studied plane waves before, the present material should help to reinforce the basic principles in the reader's mind and perhaps to introduce some concepts that the reader has not seen previously. This material will also serve as a useful reference for later chapters.

With an awareness of the historical perspective, it is usually advantageous from a pedagogical point of view to present electromagnetic theory from the "inductive," or axiomatic, approach by beginning with Maxwell's equations. The general form of time-varying Maxwell equations, then, can be written in "point," or differential, form as

$$\nabla \times \bar{\mathcal{E}} = \frac{-\partial \bar{\mathcal{B}}}{\partial t} - \bar{\mathcal{M}}, \tag{1.1a}$$

$$\nabla \times \bar{\mathcal{H}} = \frac{\partial \bar{\mathcal{D}}}{\partial t} + \bar{\mathcal{J}}, \tag{1.1b}$$

$$\nabla \cdot \bar{\mathcal{D}} = \rho, \tag{1.1c}$$

$$\nabla \cdot \bar{\mathcal{B}} = 0. \tag{1.1d}$$

The MKS system of units is used throughout this book. The script quantities represent time-varying vector fields and are real functions of spatial coordinates x, y, z, and the time variable t. These quantities are defined as follows:

$\bar{\mathcal{E}}$ is the electric field, in volts per meter (V/m).[1]

$\bar{\mathcal{H}}$ is the magnetic field, in amperes per meter (A/m).

[1] As recommended by the *IEEE Standard Definitions of Terms for Radio Wave Propagation, IEEE Standard 211-1997*, the terms "electric field" and "magnetic field" are used in place of the older terminology of "electric field intensity" and "magnetic field intensity."

$\bar{\mathcal{D}}$ is the electric flux density, in coulombs per meter squared (Coul/m^2).
$\bar{\mathcal{B}}$ is the magnetic flux density, in webers per meter squared (Wb/m^2).
$\bar{\mathcal{M}}$ is the (fictitious) magnetic current density, in volts per meter (V/m^2).
$\bar{\mathcal{J}}$ is the electric current density, in amperes per meter squared (A/m^2).
ρ is the electric charge density, in coulombs per meter cubed (Coul/m^3).

The sources of the electromagnetic field are the currents $\bar{\mathcal{M}}$ and $\bar{\mathcal{J}}$ and the electric charge density ρ. The magnetic current $\bar{\mathcal{M}}$ is a fictitious source in the sense that it is only a mathematical convenience: the real source of a magnetic current is always a loop of electric current or some similar type of magnetic dipole, as opposed to the flow of an actual magnetic charge (magnetic monopole charges are not known to exist). The magnetic current is included here for completeness, as we will have occasion to use it in Chapter 4 when dealing with apertures. Since electric current is really the flow of charge, it can be said that the electric charge density ρ is the ultimate source of the electromagnetic field.

In free-space, the following simple relations hold between the electric and magnetic field intensities and flux densities:

$$\bar{\mathcal{B}} = \mu_0 \bar{\mathcal{H}}, \tag{1.2a}$$

$$\bar{\mathcal{D}} = \epsilon_0 \bar{\mathcal{E}}, \tag{1.2b}$$

where $\mu_0 = 4\pi \times 10^{-7}$ henry/m is the permeability of free-space, and $\epsilon_0 = 8.854 \times 10^{-12}$ farad/m is the permittivity of free-space. We will see in the next section how media other than free-space affect these constitutive relations.

Equations (1.1a)–(1.1d) are linear but are not independent of each other. For instance, consider the divergence of (1.1a). Since the divergence of the curl of any vector is zero [vector identity (B.12), from Appendix B], we have

$$\nabla \cdot \nabla \times \bar{\mathcal{E}} = 0 = -\frac{\partial}{\partial t}(\nabla \cdot \bar{\mathcal{B}}) - \nabla \cdot \bar{\mathcal{M}}.$$

Since there is no free magnetic charge, $\nabla \cdot \bar{\mathcal{M}} = 0$, which leads to $\nabla \cdot \bar{\mathcal{B}} = 0$, or (1.1d). The *continuity equation* can be similarly derived by taking the divergence of (1.1b), giving

$$\nabla \cdot \bar{\mathcal{J}} + \frac{\partial \rho}{\partial t} = 0, \tag{1.3}$$

where (1.1c) was used. This equation states that charge is conserved, or that current is continuous, since $\nabla \cdot \bar{\mathcal{J}}$ represents the outflow of current at a point, and $\partial\rho/\partial t$ represents the charge buildup with time at the same point. It is this result that led Maxwell to the conclusion that the displacement current density $\partial\bar{\mathcal{D}}/\partial t$ was necessary in (1.1b), which can be seen by taking the divergence of this equation.

The above differential equations can be converted to integral form through the use of various vector integral theorems. Thus, applying the divergence theorem (B.15) to (1.1c) and (1.1d) yields

$$\oint_S \bar{\mathcal{D}} \cdot d\bar{s} = \int_V \rho \, dv = Q, \tag{1.4}$$

$$\oint_S \bar{\mathcal{B}} \cdot d\bar{s} = 0, \tag{1.5}$$

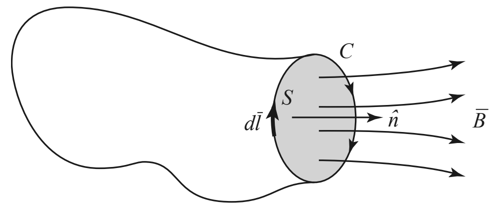


FIGURE 1.3 The closed contour C and surface S associated with Faraday's law.

where Q in (1.4) represents the total charge contained in the closed volume V (enclosed by a closed surface S). Applying Stokes' theorem (B.16) to (1.1a) gives

$$\oint_C \bar{\mathcal{E}} \cdot d\bar{l} = -\frac{\partial}{\partial t}\int_S \bar{\mathcal{B}} \cdot d\bar{s} - \int_S \bar{\mathcal{M}} \cdot d\bar{s}, \tag{1.6}$$

which, without the $\bar{\mathcal{M}}$ term, is the usual form of *Faraday's law* and forms the basis for *Kirchhoff's voltage law*. In (1.6), C represents a closed contour around the surface S, as shown in Figure 1.3. *Ampere's law* can be derived by applying Stokes' theorem to (1.1b):

$$\oint_C \bar{\mathcal{H}} \cdot d\bar{l} = \frac{\partial}{\partial t}\int_S \bar{\mathcal{D}} \cdot d\bar{s} + \int_S \bar{\mathcal{J}} \cdot d\bar{s} = \frac{\partial}{\partial t}\int_S \bar{\mathcal{D}} \cdot d\bar{s} + \mathcal{I}, \tag{1.7}$$

where $\mathcal{I} = \int_S \bar{\mathcal{J}} \cdot d\bar{s}$ is the total electric current flow through the surface S. Equations (1.4)–(1.7) constitute the integral forms of Maxwell's equations.

The above equations are valid for arbitrary time dependence, but most of our work will be involved with fields having a sinusoidal, or harmonic, time dependence, with steady-state conditions assumed. In this case phasor notation is very convenient, and so all field quantities will be assumed to be complex vectors with an implied $e^{j\omega t}$ time dependence and written with roman (rather than script) letters. Thus, a sinusoidal electric field polarized in the $\hat{x}$ direction of the form

$$\bar{\mathcal{E}}(x, y, z, t) = \hat{x}A(x, y, z)\cos(\omega t + \phi), \tag{1.8}$$

where A is the (real) amplitude, ω is the radian frequency, and ϕ is the phase reference of the wave at $t = 0$, has the phasor for

$$\bar{E}(x, y, z) = \hat{x}A(x, y, z)e^{j\phi}. \tag{1.9}$$

We will assume cosine-based phasors in this book, so the conversion from phasor quantities to real time-varying quantities is accomplished by multiplying the phasor by $e^{j\omega t}$ and taking the real part:

$$\bar{\mathcal{E}}(x, y, z, t) = \text{Re}\{\bar{E}(x, y, z)e^{j\omega t}\}, \tag{1.10}$$

as substituting (1.9) into (1.10) to obtain (1.8) demonstrates. When working in phasor notation, it is customary to suppress the factor $e^{j\omega t}$ that is common to all terms.

When dealing with power and energy we will often be interested in the time average of a quadratic quantity. This can be found very easily for time harmonic fields. For example, the average of the square of the magnitude of an electric field, given as

$$\bar{\mathcal{E}} = \hat{x}E_1\cos(\omega t + \phi_1) + \hat{y}E_2\cos(\omega t + \phi_2) + \hat{z}E_2\cos(\omega t + \phi_3), \tag{1.11}$$

has the phasor form

$$\bar{E} = \hat{x}E_1e^{j\phi_1} + \hat{y}E_2e^{j\phi_2} + \hat{z}E_3e^{j\phi_3}, \tag{1.12}$$

can be calculated as

$$
\begin{aligned}
|\bar{\mathcal{E}}|^2_{\text{avg}} &= \frac{1}{T}\int_0^T \bar{\mathcal{E}}\cdot\bar{\mathcal{E}}\,dt \\
&= \frac{1}{T}\int_0^T \left[E_1^2\cos^2(\omega t+\phi_1)+E_2^2\cos^2(\omega t+\phi_2)+E_3^2\cos^2(\omega t+\phi_3)\right]dt \\
&= \frac{1}{2}\left(E_1^2+E_2^2+E_3^2\right) = \frac{1}{2}|\bar{E}|^2 = \frac{1}{2}\bar{E}\cdot\bar{E}^*. \qquad (1.13)
\end{aligned}
$$

Then the root-mean-square (rms) value is $|\bar{E}|_{\text{rms}} = |\bar{E}|/\sqrt{2}$.

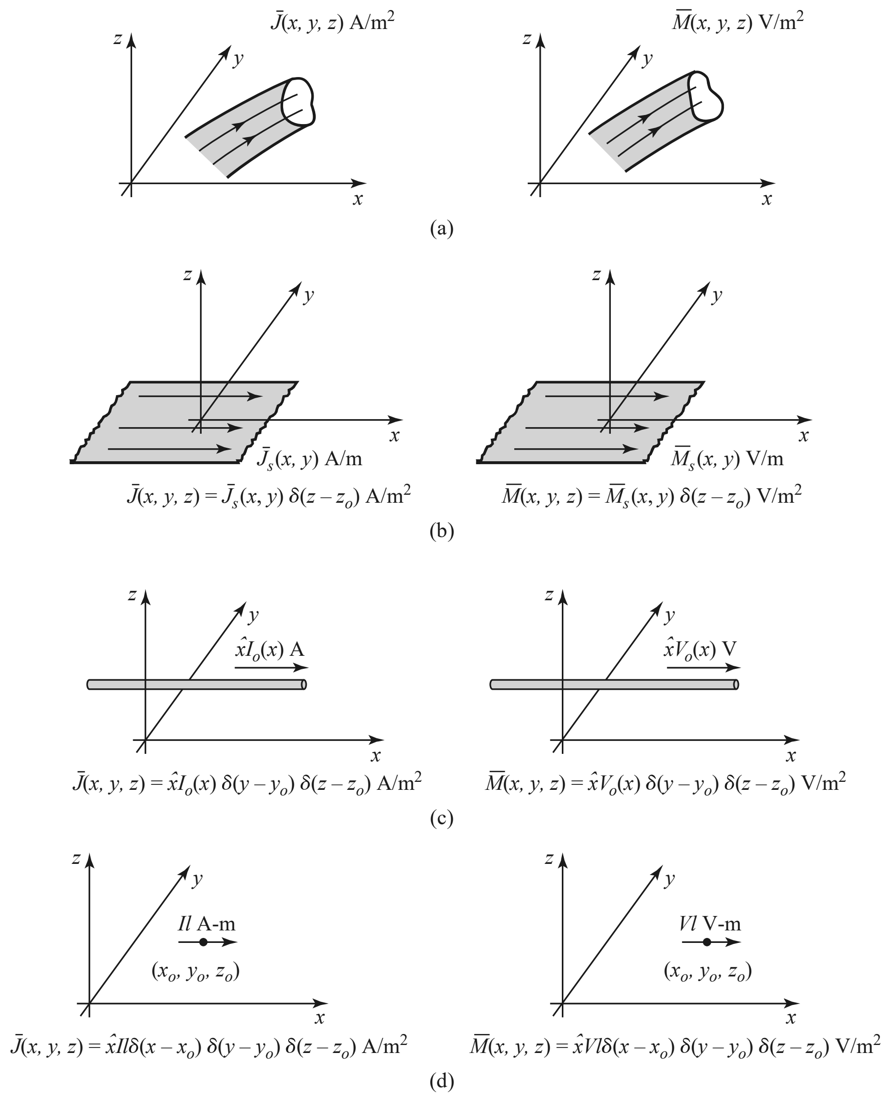


FIGURE 1.4 Arbitrary volume, surface, and line currents. (a) Arbitrary electric and magnetic volume current densities. (b) Arbitrary electric and magnetic surface current densities in the $z = z_0$ plane. (c) Arbitrary electric and magnetic line currents. (d) Infinitesimal electric and magnetic dipoles parallel to the x-axis.

Assuming an $e^{j\omega t}$ time dependence, we can replace the time derivatives in (1.1a)–(1.1d) with $j\omega$. Maxwell's equations in phasor form then become

$$\nabla \times \bar{E} = -j\omega\bar{B} - \bar{M}, \tag{1.14a}$$

$$\nabla \times \bar{H} = j\omega\bar{D} + \bar{J}, \tag{1.14b}$$

$$\nabla \cdot \bar{D} = \rho, \tag{1.14c}$$

$$\nabla \cdot \bar{B} = 0. \tag{1.14d}$$

The Fourier transform can be used to convert a solution to Maxwell's equations for an arbitrary frequency ω into a solution for arbitrary time dependence.

The electric and magnetic current sources, $\bar{J}$ and $\bar{M}$, in (1.14) are volume current densities with units A/m^2 and V/m^2. In many cases, however, the actual currents will be in the form of a current sheet, a line current, or an infinitesimal dipole current. These special types of current distributions can always be written as volume current densities through the use of delta functions. Figure 1.4 shows examples of this procedure for electric and magnetic currents.

1.3 FIELDS IN MEDIA AND BOUNDARY CONDITIONS

In the preceding section it was assumed that the electric and magnetic fields were in free-space, with no material bodies present. In practice, material bodies are often present; this complicates the analysis but also allows the useful application of material properties to microwave components. When electromagnetic fields exist in material media, the field vectors are related to each other by the constitutive relations.

For a dielectric material, an applied electric field $\bar{E}$ causes the polarization of the atoms or molecules of the material to create electric dipole moments that augment the total displacement flux, $\bar{D}$. This additional polarization vector is called $\bar{P}_e$, the *electric polarization*, where

$$\bar{D} = \epsilon_0\bar{E} + \bar{P}_e. \tag{1.15}$$

In a linear medium the electric polarization is linearly related to the applied electric field as

$$\bar{P}_e = \epsilon_0\chi_e\bar{E}, \tag{1.16}$$

where χ_e, which may be complex, is called the *electric susceptibility*. Then,

$$\bar{D} = \epsilon_0\bar{E} + \bar{P}_e = \epsilon_0(1 + \chi_e)\bar{E} = \epsilon\bar{E}, \tag{1.17}$$

where

$$\epsilon = \epsilon' - j\epsilon'' = \epsilon_0(1 + \chi_e) \tag{1.18}$$

is the complex permittivity of the medium. The imaginary part of ϵ accounts for loss in the medium (heat) due to damping of the vibrating dipole moments. (Free-space, having a real ϵ, is lossless.) Due to energy conservation, as we will see in Section 1.6, the imaginary part of ϵ must be negative (ϵ'' positive). The loss of a dielectric material may also be considered as an equivalent conductor loss. In a material with conductivity σ, a conduction current density will exist:

$$\bar{J} = \sigma\bar{E}, \tag{1.19}$$

which is *Ohm's law* from an electromagnetic field point of view. Maxwell's curl equation for $\bar{H}$ in (1.14b) then becomes

$$\begin{aligned}\nabla \times \bar{H} &= j\omega\bar{D} + \bar{J}\\ &= j\omega\epsilon\bar{E} + \sigma\bar{E}\\ &= j\omega\epsilon'\bar{E} + (\omega\epsilon'' + \sigma)\bar{E}\\ &= j\omega\left(\epsilon' - j\epsilon'' - j\frac{\sigma}{\omega}\right)\bar{E},\end{aligned} \tag{1.20}$$

where it is seen that loss due to dielectric damping ($\omega\epsilon''$) is indistinguishable from conductivity loss (σ). The term $\omega\epsilon'' + \sigma$ can then be considered as the total effective conductivity. A related quantity of interest is the *loss tangent*, defined as

$$\tan\delta = \frac{\omega\epsilon'' + \sigma}{\omega\epsilon'}, \tag{1.21}$$

which is seen to be the ratio of the real to the imaginary part of the total displacement current. Microwave materials are usually characterized by specifying the real relative permittivity (the *dielectric constant*),[2] ϵ_r, with $\epsilon' = \epsilon_r\epsilon_0$, and the loss tangent at a certain frequency. These properties are listed in Appendix G for several types of materials. It is useful to note that, after a problem has been solved assuming a lossless dielectric, loss can easily be introduced by replacing the real ϵ with a complex $\epsilon = \epsilon' - j\epsilon'' = \epsilon'(1 - j\tan\delta) = \epsilon_0\epsilon_r(1 - j\tan\delta)$.

In the preceding discussion it was assumed that $\bar{P}_e$ was a vector in the same direction as $\bar{E}$. Such materials are called *isotropic* materials, but not all materials have this property. Some materials are *anisotropic* and are characterized by a more complicated relation between $\bar{P}_e$ and $\bar{E}$, or $\bar{D}$ and $\bar{E}$. The most general linear relation between these vectors takes the form of a tensor of rank two (a dyad), which can be written in matrix form as

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = [\epsilon] \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}. \tag{1.22}$$

It is thus seen that a given vector component of $\bar{E}$ gives rise, in general, to three components of $\bar{D}$. Crystal structures and ionized gases are examples of anisotropic dielectrics. For a linear isotropic material, the matrix of (1.22) reduces to a diagonal matrix with elements ϵ.

An analogous situation occurs for magnetic materials. An applied magnetic field may align magnetic dipole moments in a magnetic material to produce a *magnetic polarization* (or magnetization) $\bar{P}_m$. Then,

$$\bar{B} = \mu_0(\bar{H} + \bar{P}_m). \tag{1.23}$$

For a linear magnetic material, $\bar{P}_m$ is linearly related to $\bar{H}$ as

$$\bar{P}_m = \chi_m\bar{H}, \tag{1.24}$$

where χ_m is a complex *magnetic susceptibility*. From (1.23) and (1.24),

$$\bar{B} = \mu_0(1 + \chi_m)\bar{H} = \mu\bar{H}, \tag{1.25}$$

[2] The *IEEE Standard Definitions of Terms for Radio Wave Propagation, IEEE Standard 211-1997,* suggests that the term "relative permittivity" be used instead of "dielectric constant." The *IEEE Standard Definitions of Terms for Antennas, IEEE Standard 145-1993*, however, still recognizes "dielectric constant." Since this term is commonly used in microwave engineering work, it will occasionally be used in this book.

where $\mu = \mu_0(1 + \chi_m) = \mu' - j\mu''$ is the complex permeability of the medium. Again, the imaginary part of χ_m or μ accounts for loss due to damping forces; there is no magnetic conductivity because there is no real magnetic current. As in the electric case, magnetic materials may be anisotropic, in which case a tensor permeability can be written as

$$\begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} \mu_{xx} & \mu_{xy} & \mu_{xz} \\ \mu_{yx} & \mu_{yy} & \mu_{yz} \\ \mu_{zx} & \mu_{zy} & \mu_{zz} \end{bmatrix} \begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix} = [\mu] \begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix}. \tag{1.26}$$

An important example of anisotropic magnetic materials in microwave engineering is the class of ferrimagnetic materials known as *ferrites*; these materials and their applications will be discussed further in Chapter 9.

If linear media are assumed (ϵ, μ not depending on $\bar{E}$ or $\bar{H}$), then Maxwell's equations can be written in phasor form as

$$\nabla \times \bar{E} = -j\omega\mu\bar{H} - \bar{M}, \tag{1.27a}$$

$$\nabla \times \bar{H} = j\omega\epsilon\bar{E} + \bar{J}, \tag{1.27b}$$

$$\nabla \cdot \bar{D} = \rho, \tag{1.27c}$$

$$\nabla \cdot \bar{B} = 0. \tag{1.27d}$$

The constitutive relations are

$$\bar{D} = \epsilon\bar{E}, \tag{1.28a}$$

$$\bar{B} = \mu\bar{H}, \tag{1.28b}$$

where ϵ and μ may be complex and may be tensors. Note that relations like (1.28a) and (1.28b) generally cannot be written in time domain form, even for linear media, because of the possible phase shift between $\bar{D}$ and $\bar{E}$, or $\bar{B}$ and $\bar{H}$. The phasor representation accounts for this phase shift by the complex form of ϵ and μ.

Maxwell's equations (1.27a)–(1.27d) in differential form require known boundary values for a complete and unique solution. A general method used throughout this book is to solve the source-free Maxwell equations in a certain region to obtain solutions with unknown coefficients and then apply boundary conditions to solve for these coefficients. A number of specific cases of boundary conditions arise, as discussed in what follows.

Fields at a General Material Interface

Consider a plane interface between two media, as shown in Figure 1.5. Maxwell's equations in integral form can be used to deduce conditions involving the normal and tangential

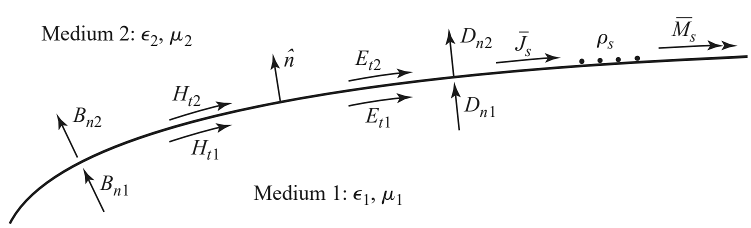


FIGURE 1.5 Fields, currents, and surface charge at a general interface between two media.

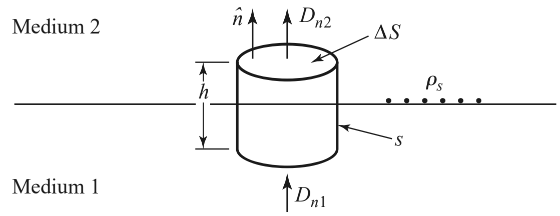


FIGURE 1.6 Closed surface S for equation (1.29).

fields at this interface. The time-harmonic version of (1.4), where S is the closed "pillbox"-shaped surface shown in Figure 1.6, can be written as

$$\oint_S \bar{D} \cdot d\bar{s} = \int_V \rho \, dv. \tag{1.29}$$

In the limit as $h \to 0$, the contribution of D_{tan} through the sidewalls goes to zero, so (1.29) reduces to

$$\Delta S D_{2n} - \Delta S D_{1n} = \Delta S \rho_s,$$

or

$$D_{2n} - D_{1n} = \rho_s, \tag{1.30}$$

where ρ_s is the surface charge density on the interface. In vector form, we can write

$$\hat{n} \cdot (\bar{D}_2 - \bar{D}_1) = \rho_s. \tag{1.31}$$

A similar argument for $\bar{B}$ leads to the result that

$$\hat{n} \cdot \bar{B}_2 = \hat{n} \cdot \bar{B}_1, \tag{1.32}$$

because there is no free magnetic charge.

For the tangential components of the electric field we use the phasor form of (1.6),

$$\oint_C \bar{E} \cdot d\bar{l} = -j\omega \int_S \bar{B} \cdot d\bar{s} - \int_S \bar{M} \cdot d\bar{s}, \tag{1.33}$$

in connection with the closed contour C shown in Figure 1.7. In the limit as $h \to 0$, the surface integral of $\bar{B}$ vanishes (because $S = h\Delta\ell$ vanishes). The contribution from the surface integral of $\bar{M}$, however, may be nonzero if a magnetic surface current density $\bar{M}_s$ exists on the surface. The Dirac delta function can then be used to write

$$\bar{M} = \bar{M}_s \delta(h), \tag{1.34}$$

where h is a coordinate measured normal from the interface. Equation (1.33) then gives

$$\Delta\ell E_{t1} - \Delta\ell E_{t2} = -\Delta\ell M_s,$$

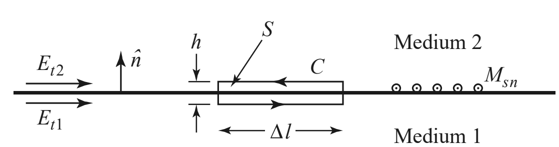


FIGURE 1.7 Closed contour C for equation (1.33).

or

$$E_{t1} - E_{t2} = -M_s, \tag{1.35}$$

which can be generalized in vector form as

$$(\bar{E}_2 - \bar{E}_1) \times \hat{n} = \bar{M}_s. \tag{1.36}$$

A similar argument for the magnetic field leads to

$$\hat{n} \times (\bar{H}_2 - \bar{H}_1) = \bar{J}_s, \tag{1.37}$$

where $\bar{J}_s$ is an electric surface current density that may exist at the interface. Equations (1.31), (1.32), (1.36), and (1.37) are the most general expressions for the boundary conditions at an arbitrary interface of materials and/or surface currents.

Fields at a Dielectric Interface

At an interface between two lossless dielectric materials, no charge or surface current densities will ordinarily exist. Equations (1.31), (1.32), (1.36), and (1.37) then reduce to

$$\hat{n} \cdot \bar{D}_1 = \hat{n} \cdot \bar{D}_2, \tag{1.38a}$$

$$\hat{n} \cdot \bar{B}_1 = \hat{n} \cdot \bar{B}_2, \tag{1.38b}$$

$$\hat{n} \times \bar{E}_1 = \hat{n} \times \bar{E}_2, \tag{1.38c}$$

$$\hat{n} \times \bar{H}_1 = \hat{n} \times \bar{H}_2. \tag{1.38d}$$

In words, these equations state that the normal components of $\bar{D}$ and $\bar{B}$ are continuous across the interface, and the tangential components of $\bar{E}$ and $\bar{H}$ are continuous across the interface. Because Maxwell's equations are not all linearly independent, the six boundary conditions contained in the above equations are not all linearly independent. Thus, the enforcement of (1.38c) and (1.38d) for the four tangential field components, for example, will automatically force the satisfaction of the equations for the continuity of the normal components.

Fields at the Interface with a Perfect Conductor (Electric Wall)

Many problems in microwave engineering involve boundaries with good conductors (e.g., metals), which can often be assumed as lossless ($\sigma \to \infty$). In this case of a perfect conductor, all field components must be zero inside the conducting region. This result can be seen by considering a conductor with finite conductivity ($\sigma < \infty$) and noting that the skin depth (the depth to which most of the microwave power penetrates) goes to zero as $\sigma \to \infty$. (Such an analysis will be performed in Section 1.7.) If we also assume here that $\bar{M}_s = 0$, which would be the case if the perfect conductor filled all the space on one side of the boundary, then (1.31), (1.32), (1.36), and (1.37) reduce to the following:

$$\hat{n} \cdot \bar{D} = \rho_s, \tag{1.39a}$$

$$\hat{n} \cdot \bar{B} = 0, \tag{1.39b}$$

$$\hat{n} \times \bar{E} = 0, \tag{1.39c}$$

$$\hat{n} \times \bar{H} = \bar{J}_s, \tag{1.39d}$$

where ρ_s and $\bar{J}_s$ are the electric surface charge density and current density, respectively, on the interface, and $\hat{n}$ is the normal unit vector pointing out of the perfect conductor. Such a boundary is also known as an *electric wall* because the tangential components of $\bar{E}$ are "shorted out," as seen from (1.39c), and must vanish at the surface of the conductor.

The Magnetic Wall Boundary Condition

Dual to the preceding boundary condition is the *magnetic wall* boundary condition, where the tangential components of $\bar{H}$ must vanish. Such a boundary does not really exist in practice but may be approximated by a corrugated surface or in certain planar transmission line problems. In addition, the idealization that $\hat{n} \times \bar{H} = 0$ at an interface is often a convenient simplification, as we will see in later chapters. We will also see that the magnetic wall boundary condition is analogous to the relations between the voltage and current at the end of an open-circuited transmission line, while the electric wall boundary condition is analogous to the voltage and current at the end of a short-circuited transmission line. The magnetic wall condition, then, provides a degree of completeness in our formulation of boundary conditions and is a useful approximation in several cases of practical interest.

The fields at a magnetic wall satisfy the following conditions:

$$\hat{n} \cdot \bar{D} = 0, \tag{1.40a}$$

$$\hat{n} \cdot \bar{B} = 0, \tag{1.40b}$$

$$\hat{n} \times \bar{E} = -\bar{M}_s, \tag{1.40c}$$

$$\hat{n} \times \bar{H} = 0, \tag{1.40d}$$

where $\hat{n}$ is the normal unit vector pointing out of the magnetic wall region.

The Radiation Condition

When dealing with problems that have one or more infinite boundaries, such as plane waves in an infinite medium, or infinitely long transmission lines, a condition on the fields at infinity must be enforced. This boundary condition is known as the *radiation condition* and is essentially a statement of energy conservation. It states that, at an infinite distance from a source, the fields must either be vanishingly small (i.e., zero) or propagating in an outward direction. This result can easily be seen by allowing the infinite medium to contain a small loss factor (as any physical medium would have). Incoming waves (from infinity) of finite amplitude would then require an infinite source at infinity and so are disallowed.

1.4 THE WAVE EQUATION AND BASIC PLANE WAVE SOLUTIONS

The Helmholtz Equation

In a source-free, linear, isotropic, homogeneous region, Maxwell's curl equations in phasor form are

$$\nabla \times \bar{E} = -j\omega\mu\bar{H}, \tag{1.41a}$$

$$\nabla \times \bar{H} = j\omega\epsilon\bar{E}, \tag{1.41b}$$

and constitute two equations for the two unknowns, $\bar{E}$ and $\bar{H}$. As such, they can be solved for either $\bar{E}$ or $\bar{H}$. Taking the curl of (1.41a) and using (1.41b) gives

$$\nabla \times \nabla \times \bar{E} = -j\omega\mu\nabla \times \bar{H} = \omega^2\mu\epsilon\bar{E},$$

which is an equation for $\bar{E}$. This result can be simplified through the use of vector identity (B.14), $\nabla \times \nabla \times \bar{A} = \nabla(\nabla \cdot \bar{A}) - \nabla^2 \bar{A}$, which is valid for the rectangular components of an arbitrary vector $\bar{A}$. Then,

$$\nabla^2 \bar{E} + \omega^2 \mu\epsilon \bar{E} = 0, \tag{1.42}$$

because $\nabla \cdot \bar{E} = 0$ in a source-free region. Equation (1.42) is the *wave equation*, or *Helmholtz equation*, for $\bar{E}$. An identical equation for $\bar{H}$ can be derived in the same manner:

$$\nabla^2 \bar{H} + \omega^2 \mu\epsilon \bar{H} = 0. \tag{1.43}$$

A constant $k = \omega\sqrt{\mu\epsilon}$ is defined and called the *propagation constant* (also known as the *phase constant*, or *wave number*), of the medium; its units are 1/m.

As a way of introducing wave behavior, we will next study the solutions to the above wave equations in their simplest forms, first for a lossless medium and then for a lossy (conducting) medium.

Plane Waves in a Lossless Medium

In a lossless medium, ϵ and μ are real numbers, and so k is real. A basic plane wave solution to the above wave equations can be found by considering an electric field with only an $\hat{x}$ component and uniform (no variation) in the x and y directions. Then, $\partial/\partial x = \partial/\partial y = 0$, and the Helmholtz equation of (1.42) reduces to

$$\frac{\partial^2 E_x}{\partial z^2} + k^2 E_x = 0. \tag{1.44}$$

The two independent solutions to this equation are easily seen, by substitution, to be of the form

$$E_x(z) = E^+ e^{-jkz} + E^- e^{jkz}, \tag{1.45}$$

where E^+ and E^- are arbitrary amplitude constants.

The above solution is for the time harmonic case at frequency ω. In the time domain, this result is written as

$$\mathcal{E}_x(z, t) = E^+ \cos(\omega t - kz) + E^- \cos(\omega t + kz), \tag{1.46}$$

where we have assumed that E^+ and E^- are real constants. Consider the first term in (1.46). This term represents a wave traveling in the $+z$ direction because, to maintain a fixed point on the wave ($\omega t - kz =$ constant), one must move in the $+z$ direction as time increases. Similarly, the second term in (1.46) represents a wave traveling in the negative z direction—hence the notation E^+ and E^- for these wave amplitudes. The velocity of the wave in this sense is called the *phase velocity* because it is the velocity at which a fixed phase point on the wave travels, and it is given by

$$v_p = \frac{dz}{dt} = \frac{d}{dt}\left(\frac{\omega t - \text{constant}}{k}\right) = \frac{\omega}{k} = \frac{1}{\sqrt{\mu\epsilon}} \tag{1.47}$$

In free-space, we have $v_p = 1/\sqrt{\mu_0\epsilon_0} = c = 2.998 \times 10^8$ m/sec, which is the speed of light.

The *wavelength*, λ, is defined as the distance between two successive maxima (or minima, or any other reference points) on the wave at a fixed instant of time. Thus,

$$(\omega t - kz) - [\omega t - k(z + \lambda)] = 2\pi,$$

so

$$\lambda = \frac{2\pi}{k} = \frac{2\pi v_p}{\omega} = \frac{v_p}{f}. \tag{1.48}$$

A complete specification of the plane wave electromagnetic field should include the magnetic field. In general, whenever $\bar{E}$ or $\bar{H}$ is known, the other field vector can be readily found by using one of Maxwell's curl equations. Thus, applying (1.41a) to the electric field of (1.45) gives $H_x = H_z = 0$, and

$$H_y = \frac{j}{\omega\mu}\frac{\partial E_x}{\partial z} = \frac{1}{\eta}(E^+ e^{-jkz} - E^- e^{jkz}), \tag{1.49}$$

where $\eta = \omega\mu/k = \sqrt{\mu/\epsilon}$ is known as the *intrinsic impedance* of the medium. The ratio of the $\bar{E}$ and $\bar{H}$ field components is seen to have units of impedance, known as the *wave impedance*; for planes waves the wave impedance is equal to the intrinsic impedance of the medium. In free-space the intrinsic impedance is $\eta_0 = \sqrt{\mu_0/\epsilon_0} = 377\ \Omega$. Note that the $\bar{E}$ and $\bar{H}$ vectors are orthogonal to each other and orthogonal to the direction of propagation ($\pm\hat{z}$); this is a characteristic of transverse electromagnetic (TEM) waves.

EXAMPLE 1.1 BASIC PLANE WAVE PARAMETERS

A plane wave propagating in a lossless dielectric medium has an electric field given as $\mathcal{E}_x = E_0 \cos(\omega t - \beta z)$ with a frequency of 5.0 GHz and a wavelength in the material of 3.0 cm. Determine the propagation constant, the phase velocity, the relative permittivity of the medium, and the wave impedance.

Solution

From (1.48) the propagation constant is $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{0.03} = 209.4\ \text{m}^{-1}$, and from (1.47) the phase velocity is

$$v_p = \frac{\omega}{k} = \frac{2\pi f}{k} = \lambda f = (0.03)\,(5 \times 10^9) = 1.5 \times 10^8\ \text{m/sec}.$$

This is slower than the speed of light by a factor of 2.0. The relative permittivity of the medium can be found from (1.47) as

$$\epsilon_r = \left(\frac{c}{v_p}\right)^2 = \left(\frac{3.0 \times 10^8}{1.5 \times 10^8}\right)^2 = 4.0$$

The wave impedance is

$$\eta = \eta_0/\sqrt{\epsilon_r} = \frac{377}{\sqrt{4.0}} = 188.5\ \Omega \qquad \blacksquare$$

Plane Waves in a General Lossy Medium

Now consider the effect of a lossy medium. If the medium is conductive, with a conductivity σ, Maxwell's curl equations can be written, from (1.41a) and (1.20) as

$$\nabla \times \bar{E} = -j\omega\mu\bar{H}, \tag{1.50a}$$

$$\nabla \times \bar{H} = j\omega\epsilon\bar{E} + \sigma\bar{E}. \tag{1.50b}$$

The resulting wave equation for $\bar{E}$ then becomes

$$\nabla^2 \bar{E} + \omega^2 \mu\epsilon \left(1 - j\frac{\sigma}{\omega\epsilon}\right) \bar{E} = 0, \tag{1.51}$$

where we see a similarity with (1.42), the wave equation for $\bar{E}$ in the lossless case. The difference is that the quantity $k^2 = \omega^2\mu\epsilon$ of (1.42) is replaced by $\omega^2\mu\epsilon[1 - j(\sigma/\omega\epsilon)]$ in (1.51). We then define a *complex propagation constant* for the medium as

$$\gamma = \alpha + j\beta = j\omega\sqrt{\mu\epsilon}\sqrt{1 - j\frac{\sigma}{\omega\epsilon}} \tag{1.52}$$

where α is the *attenuation constant* and β is the *phase constant.* If we again assume an electric field with only an $\hat{x}$ component and uniform in x and y, the wave equation of (1.51) reduces to

$$\frac{\partial^2 E_x}{\partial z^2} - \gamma^2 E_x = 0, \tag{1.53}$$

which has solutions

$$E_x(z) = E^+ e^{-\gamma z} + E^- e^{\gamma z}. \tag{1.54}$$

The positive traveling wave then has a propagation factor of the form

$$e^{-\gamma z} = e^{-\alpha z} e^{-j\beta z},$$

which in the time domain is of the form

$$e^{-\alpha z}\cos(\omega t - \beta z).$$

We see that this represents a wave traveling in the $+z$ direction with a phase velocity $v_p = \omega/\beta$, a wavelength $\lambda = 2\pi/\beta$, and an exponential damping factor. The rate of decay with distance is given by the attenuation constant, α. The negative traveling wave term of (1.54) is similarly damped along the $-z$ axis. If the loss is removed, $\sigma = 0$, and we have $\gamma = jk$ and $\alpha = 0$, $\beta = k$.

As discussed in Section 1.3, loss can also be treated through the use of a complex permittivity. From (1.52) and (1.20) with $\sigma = 0$ but $\epsilon = \epsilon' - j\epsilon''$ complex, we have that

$$\gamma = j\omega\sqrt{\mu\epsilon} = jk = j\omega\sqrt{\mu\epsilon'(1 - j\tan\delta)}, \tag{1.55}$$

where $\tan\delta = \epsilon''/\epsilon'$ is the loss tangent of the material.

The associated magnetic field can be calculated as

$$H_y = \frac{j}{\omega\mu}\frac{\partial E_x}{\partial z} = \frac{-j\gamma}{\omega\mu}(E^+ e^{-\gamma z} - E^- e^{\gamma z}). \tag{1.56}$$

The intrinsic impedance of the conducting medium is now complex,

$$\eta = \frac{j\omega\mu}{\gamma}, \tag{1.57}$$

but is still identified as the wave impedance, which expresses the ratio of electric to magnetic field components. This allows (1.56) to be rewritten as

$$H_y = \frac{1}{\eta}(E^+ e^{-\gamma z} - E^- e^{\gamma z}). \tag{1.58}$$

Note that although η of (1.57) is, in general, complex, it reduces to the lossless case of $\eta = \sqrt{\mu/\epsilon}$ when $\gamma = jk = j\omega\sqrt{\mu\epsilon}$.

Plane Waves in a Good Conductor

Many problems of practical interest involve loss or attenuation due to good (but not perfect) conductors. A good conductor is a special case of the preceding analysis, where the conductive current is much greater than the displacement current, which means that $\sigma \gg \omega\epsilon$. Most metals can be categorized as good conductors. In terms of a complex ϵ, rather than conductivity, this condition is equivalent to $\epsilon'' \gg \epsilon'$. The propagation constant of (1.52) can then be adequately approximated by ignoring the displacement current term, to give

$$\gamma = \alpha + j\beta \simeq j\omega\sqrt{\mu\epsilon}\sqrt{\frac{\sigma}{j\omega\epsilon}} = (1+j)\sqrt{\frac{\omega\mu\sigma}{2}}. \tag{1.59}$$

The *skin depth*, or characteristic depth of penetration, is defined as

$$\delta_s = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\sigma}}. \tag{1.60}$$

Thus the amplitude of the fields in the conductor will decay by an amount $1/e$, or 36.8%, after traveling a distance of one skin depth, because $e^{-\alpha z} = e^{-\alpha\delta_s} = e^{-1}$. At microwave frequencies, for a good conductor, this distance is very small. The practical importance of this result is that only a thin plating of a good conductor (e.g., silver or gold) is necessary for low-loss microwave components.

EXAMPLE 1.2 SKIN DEPTH AT MICROWAVE FREQUENCIES

Compute the skin depth of aluminum, copper, gold, and silver at a frequency of 10 GHz.

Solution

The conductivities for these metals are listed in Appendix F. Equation (1.60) gives the skin depths as

$$\delta_s = \sqrt{\frac{2}{\omega\mu\sigma}} = \sqrt{\frac{1}{\pi f\mu_0\sigma}} = \sqrt{\frac{1}{\pi(10^{10})(4\pi\times 10^{-7})}}\sqrt{\frac{1}{\sigma}}$$
$$= 5.03\times 10^{-3}\sqrt{\frac{1}{\sigma}}.$$

For aluminum: $\delta_s = 5.03\times 10^{-3}\sqrt{\dfrac{1}{3.816\times 10^7}} = 8.14\times 10^{-7}\text{m}.$

For copper: $\delta_s = 5.03\times 10^{-3}\sqrt{\dfrac{1}{5.813\times 10^7}} = 6.60\times 10^{-7}\text{m}.$

For gold: $\delta_s = 5.03\times 10^{-3}\sqrt{\dfrac{1}{4.098\times 10^7}} = 7.86\times 10^{-7}\text{m}.$

For silver: $\delta_s = 5.03\times 10^{-3}\sqrt{\dfrac{1}{6.173\times 10^7}} = 6.40\times 10^{-7}\text{m}.$

These results show that most of the current flow in a good conductor occurs in an extremely thin region near the surface of the conductor. ■

TABLE 1.1 Summary of Results for Plane Wave Propagation in Various Media

	Type of Medium		
Quantity	Lossless ($\epsilon'' = \sigma = 0$)	General Lossy	Good Conductor ($\epsilon'' \gg \epsilon'$ or $\sigma \gg \omega\epsilon'$)
Complex propagation constant	$\gamma = j\omega\sqrt{\mu\epsilon}$	$\gamma = j\omega\sqrt{\mu\epsilon}$ $= j\omega\sqrt{\mu\epsilon'}\sqrt{1 - j\dfrac{\sigma}{\omega\epsilon'}}$	$\gamma = (1+j)\sqrt{\omega\mu\sigma/2}$
Phase constant (wave number)	$\beta = k = \omega\sqrt{\mu\epsilon}$	$\beta = \mathrm{Im}\{\gamma\}$	$\beta = \mathrm{Im}\{\gamma\} = \sqrt{\omega\mu\sigma/2}$
Attenuation constant	$\alpha = 0$	$\alpha = \mathrm{Re}\{\gamma\}$	$\alpha = \mathrm{Re}\{\gamma\} = \sqrt{\omega\mu\sigma/2}$
Impedance	$\eta = \sqrt{\mu/\epsilon} = \omega\mu/k$	$\eta = j\omega\mu/\gamma$	$\eta = (1+j)\sqrt{\omega\mu/2\sigma}$
Skin depth	$\delta_s = \infty$	$\delta_s = 1/\alpha$	$\delta_s = \sqrt{2/\omega\mu\sigma}$
Wavelength	$\lambda = 2\pi/\beta$	$\lambda = 2\pi/\beta$	$\lambda = 2\pi/\beta$
Phase velocity	$v_p = \omega/\beta$	$v_p = \omega/\beta$	$v_p = \omega/\beta$

The intrinsic impedance inside a good conductor can be obtained from (1.57) and (1.59). The result is

$$\eta = \frac{j\omega\mu}{\gamma} \simeq (1+j)\sqrt{\frac{\omega\mu}{2\sigma}} = (1+j)\frac{1}{\sigma\delta_s}. \tag{1.61}$$

Notice that the phase angle of this impedance is 45°, a characteristic of good conductors. The phase angle of the impedance for a lossless material is 0°, and the phase angle of the impedance of an arbitrary lossy medium is somewhere between 0° and 45°.

Table 1.1 summarizes the results for plane wave propagation in lossless and lossy homogeneous media.

1.5 GENERAL PLANE WAVE SOLUTIONS

Some specific features of plane waves were discussed in Section 1.4, but we will now look at plane waves from a more general point of view and solve the wave equation by the method of separation of variables. This technique will find application in succeeding chapters. We will also discuss circularly polarized plane waves, which will be important for the discussion of ferrites in Chapter 9.

In free-space, the Helmholtz equation for $\bar{E}$ can be written as

$$\nabla^2\bar{E} + k_0^2\bar{E} = \frac{\partial^2\bar{E}}{\partial x^2} + \frac{\partial^2\bar{E}}{\partial y^2} + \frac{\partial^2\bar{E}}{\partial z^2} + k_0^2\bar{E} = 0, \tag{1.62}$$

and this vector wave equation holds for each rectangular component of $\bar{E}$:

$$\frac{\partial^2 E_i}{\partial x^2} + \frac{\partial^2 E_i}{\partial y^2} + \frac{\partial^2 E_i}{\partial z^2} + k_0^2 E_i = 0, \tag{1.63}$$

where the index $i = x, y$, or z. This equation can be solved by the method of *separation of variables*, a standard technique for treating such partial differential equations. The method begins by assuming that the solution to (1.63) for, say, E_x, can be written as a product of three functions for each of the three coordinates:

$$E_x(x, y, z) = f(x)g(y)h(z). \tag{1.64}$$